Visual servo control for an underactuated system

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Abstract: This paper discusses the visual servo control of an underactuated mechanisms under fixed-camera configuration. The control goal is to stabilize the system over a desired static target by using a vision system equipped with a fixed camera to observer the system and target. We present a control scheme based on the combination of a state observer and the visual feedback is applied to an underactuated system, the so-called *Pendubot*, consisting in a double pendulum actuated only at the first joint. The paper ends with the presentation of several simulation results and some guidelines future work are drawn in the conclusion.1

1 Introduction

This paper presents the application of modern linear systems theory with visual sensor to control of underactuacted mechanical system. In the eighties, the control of robot manipulators was extensively studied. Several control strategies based on passivity, Lyapunov theory, feedback linearization, output regulation, etc. have been developed for the fully actuated case, i.e. systems with the same number of actuators as degree of freedom [6, 10, 19]. The techniques developed for fully actuated robots do not apply directly to the case of underactuated mechanical systems [3, 7, 12{21]. Underactuated mechanical systems or vehicles are systems with fewer independent control actuators than degrees of freedom to be controlled. The use of visual sensors in feedback control loops with robot manipulators represents an attractive solution to position and motion control [1, 2, 4, 5]. Most existing generalizations of classical visual servoing techniques exploit a high gain or computed torque feedback to make a

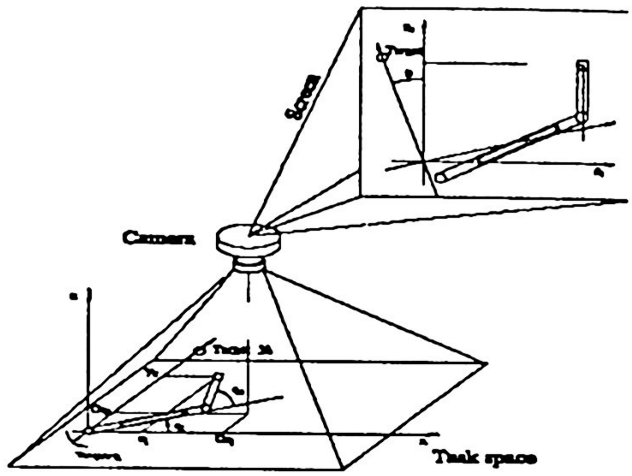

Fig. 1. Schematic representation of the Robot-camera system

dynamic reduction of the system to a controllable kinematic model for which the visual servoing task may be solved directly [5]. The dynamics model of a system is commonly ignored in the design of visual servo systems and closed-loop performance may be severely limited to ensure that the dynamic reduction is valid. Recently in [11] has explored a more nonlinear aspect of the system dynamics, and presented an asymptotically stable method for position regulation for fixed-camera visual servoing. The dificulties associated with controlling an underactuated system have received even less attention. In [22] has been working on the visual servoing problem using a Lagrangian representation of the system dynamics and consider underactuated and nonholonomic systems. In [8] proposed a new imagebased control strategy for visual servoing which is applicable to a class of underactuated dynamic systems.

In this paper we develop a dynamic controller using visual sensor for an underactuated dynamic system which operate with accurate target information as shown in Figure 1. The proposed approach is motivated by a theoretical analysis of the dynamic equation of motion of a rigid body and exploits structural linear properties of these dynamics to derive a linear control algorithm.
The paper is organized in the following manner. Section 2 describes the robot manipulator system model while section 3 is devoted to the design of the controller structure, whose performance is shown in Section 4 by presenting some simulation results. Finally, concluding remarks are given in Section 5.

2. The Pendubot Model

The Pendubot, which is the underactuated system considered here, it is shown schematically in Figure 2. For the p of this work, we assume that it has a planar motion without friction.

Table 1. PARAMETERS OF THE PENDUBOT

	notation	value	unit
Mass of link 1	m_1	0.5289	kg
Mass of link 2	m_2	0.3316	kg
Length of link 1	l_1	0.26987	m
Length of link 2	l_2	0.38117	m
Distance to the center of mass of link 1	l_{c_1}	0.13191	m
Distance to the center of mass of link 2	l_{c_2}	0.19208	m
Moment of inertia of link 1 about its centroid	I_1	0.013863	Kgm^2
Moment of inertia of link 2 about its centroid	I_2	0.016749	Kgm^2
Acceleration due to gravity	g	9.81	m/sec^2
Angle that link 1 makes with the horizontal	q_1		rad
Angle that link 2 makes with the link 1	q_2		rad
Torque applied on link 1	τ_1		$Nw \cdot m$

The equation of motion for the PENDUBOT can be described by the standard equation [19]

$$D(q)\ddot{q} + C(q,\dot{q}) + G(q) = \tau \qquad (1)$$

where q is the vector of joint variables (generalized coordinates), D(q) is the nxn inertia matrix, $C(q,\dot{q})$ is the vector of Coriolis and centripetal torques, G(q) are the gravitational terms, and τ is the vector of input torques. If only m joints are actuated, vector

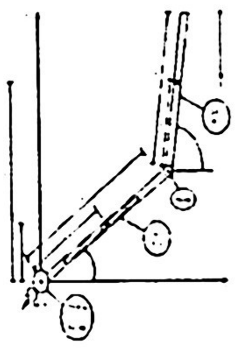

Fig. 2. The PENDUBOT system

q can be partitioned, without loss of generality as (q_1, q_2); where $q_1 \in R^m$ represents the actuated joints, and $q_1 \in R^{(n-m)}$ represents the unactuated ones. For the Pendubot system, the dynamic model (1) is particularized as

$$\begin{bmatrix} D_{11} & D_{12} \\ D_{12} & D_{22} \end{bmatrix} \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} C_1 \\ C_2 \end{bmatrix} + \begin{bmatrix} G_1 \\ G_2 \end{bmatrix} = \begin{bmatrix} \tau_1 \\ 0 \end{bmatrix} \qquad (2)$$

where

$$
\begin{aligned}
D_{11} &= m_1 l_{c1}^2 + m_2\left(l_1^2 + l_{c2}^2 + 2 l_1 l_{c2}\cos(q_2)\right) + I_1 + I_2\\
D_{12} &= m_2\left(l_{c2}^2 + l_1 l_{c2}\cos(q_2)\right) + I_2\\
D_{22} &= m_2 l_{c2}^2 + I_2\\
C_1 &= -2 m_2 l_1 l_{c2}\dot{q}_1\dot{q}_2 \sin(q_2) - m_2 l_1 l_{c2}\dot{q}_2^2 \sin(q_2)\\
C_2 &= m_2 l_1 l_{c2}\dot{q}_1^2 \sin(q_2)\\
G_1 &= m_1 g l_{c1}\cos(q_1) + m_2 g l_1 \cos(q_1) + m_2 g l_{c2}\cos(q_1 + q_2)\\
G_2 &= m_2 g l_{c2}\cos(q_1 + q_2).
\end{aligned} \tag{3}
$$

Choosing as state vector $x = (x_1 x_2 x_3 x_4)^T := (q_1 q_2 \dot{q}_1 \dot{q}_2)^T$, as input $u = \tau_1$; and q_2 as the output, the description of the system can be given in state space form as:

$$\dot{x}(t) = f(x) + g(x)u(t) \tag{4}$$

$$y(t) = h(x) \tag{5}$$

where

$$
f(x) = \begin{bmatrix} x_3 \\ x_4 \\ \frac{D_{22}}{D_{11}D_{22} - D_{12}^2}\left\{\frac{D_{12}C_2}{D_{22}} + \frac{D_{12}G_2}{D_{22}} - C_1 - G_1\right\} \\ \frac{D_{12}}{D_{11}D_{22} - D_{12}^2}\left\{\frac{D_{12}C_2}{D_{22}} + \frac{D_{12}G_2}{D_{22}} - C_1 - G_1\right\} - \frac{C_2}{D_{22}} - \frac{G_2}{D_{22}} \end{bmatrix},
$$

$$
g(x) = \begin{bmatrix} 0 \\ 0 \\ \frac{D_{22}}{D_{11}D_{22} - D_{12}^2} \\ \frac{D_{12}}{D_{11}D_{22} - D_{12}^2} \end{bmatrix}, h(x) = x_2. \tag{6}
$$

The control objective is to stabilize the system around its unstable equilibrium point $x^* = (x^*_1 x^*_2 x^*_3 x^*_4)^T = \left(\frac{\pi}{2}, 0, 0, 0\right)^T$ i.e. to bring the second pendulum to its upper position and the first angle q_1 to zero simultaneously.

2.1 Controllability and Observability of the Linearized Model

When the Pendubot is in a neighborhood of its top unstable equilibrium position, a linear controller can stabilize the pendulum quite adequately. In order to implement a visual servo linear controller, the general non-linear differential equations (4)-(5) are linearized about the top equilibrium position. Provided that the linearized system is observable and controllable, we can design a dynamic linear control. Let us therefore compute the rank of the controllability and observability matrices. Linearizing the non-linear equations (4)-(5) about the top unstable equilibrium point, we have

$$\begin{aligned} \delta\dot{x}(t) &= A\delta x(t) + B\delta u(t) \\ \delta y(t) &= C\delta x(t) \end{aligned} \tag{7}$$

Where

$$A = \frac{\partial f(x^*)}{\partial x}\Big|_{x=x^*} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 32.2199 & -10.0172 & 0 & 0 \\ -29.8052 & 37.6597 & 0 & 0 \end{bmatrix},$$

$$B = g(x^*) = \begin{bmatrix} 0 \\ 0 \\ 26.6507 \\ -12.5387 \end{bmatrix},$$

$$C = \frac{\partial h}{\partial x}\Big|_{x=x^*} = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$$

This model will be used to compute the control law necessary for achieving the tracking of a reference signal. Then, we have

$$det[B \mid AB \mid A^2B \mid A^3B] = -888.3171 \tag{8}$$

$$det[C \mid CA \mid CA^2 \mid CA^3]^T = -8.1198 \times 10^7 \tag{9}$$

The linearized system is controllable and observable. Therefore, a complete order observer and a full state feedback $u = -Kx$ with an appropriate gain vector K are able to successfully stabilize the system in a neighborhood of its unstable equilibrium point.

2.2 Discrete-Time State Space Equation

To obtain a discrete-time state-space equation from a continuoustime state-space equation (7)-(8), we use the following MATLAB command [Φ, Γ] = c2d(A,B,T) where T is the sampling period and it is equal to 0:005 sec. The state equation is

$$x(k+1) = \Phi x(k) + \Gamma u(k) \tag{10}$$

$$y(k) = Cx(k) \tag{11}$$

Where

$$\Phi = \begin{bmatrix} 1 & -0.0001252 & 0.005 & 0 \\ -0.0003726 & 1 & 0 & 0.005001 \\ 0.1613 & -0.0501 & 1 & -0.0001252 \\ -0.1191 & 0.1883 & -0.0003726 & 1 \end{bmatrix};$$

$$\Gamma = \begin{bmatrix} 0.0003332 \\ -0.0005318 \\ 0.1333 \\ -0.2127 \end{bmatrix},$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}.$$

Here, $x(k)$ is the state vector (n-vector) at kth sampling instant, $u(k)$ control signal (scalar) at kth sampling and $y(k)$ is the output at kth sampling.

3 Control Scheme

In this section it is shown how a visual servoing control may be derived based on estimation techniques for an fix camera with underactuated rigid body dynamics. Visual servoing systems incorporate the visual sensors in the feedback. Figure 3 depicts a block diagram of the closed-loop control system, this is a block diagram of one degree of freedom (1-DOF). The camera lens is modelled as a simple gain, *Klens*, which, due to perspective, is a function of target distance. The system is multi-rate due to the different sample intervals for the Pendubot's servo system and the camera's field rate. Pendubot structural dynamic effects are reprenented by $H(s)$, which is knowed. We shall first discuss the full-order state observer and then the state feedback controller.

It is important to note that, in the present analysis, state $x(k)$ is not available for direct measurement. Since the output $y(k)=Cx(k)$ can be measured by the fix camera. We have already checked that the system is completely observable and controllable. Therefore, we can design a state observer

$$\tilde{x}(k+1) = \Phi\tilde{x}(k) + \Gamma u(k) + K_e[y(k) - \tilde{y}(k)] \qquad (12)$$

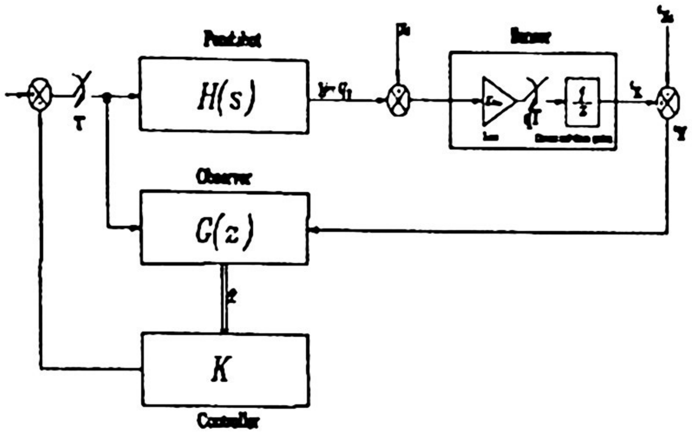

Fig. 3. Structure of visual servo control for the Pendubot. Here, x_t is the world coordinate location of the target, ${}^{i}x_d$ is the desired location of the target on the image plane, and ${}^{i}\tilde{x}={}^{i}x-{}^{i}x_d$ is the image plane error.

and the observed state $\hat{x}(k)$ is used to form the vector control $u(k)$, or

$$u(k) = -K\hat{x}(k) \tag{13}$$

where K_e and K are the observer feedback gain matrix and the state feedback gain matrix, respectively. The problem of stability analysis with multi-rate can be handled by the modified z-transform.

4 Simulation Result

In all the simulations, we consider that the initial condition of the system is near the equilibrium point x^{*} and the gain K that stabilizes

$$K = \begin{bmatrix} -22.1131 & -21.2982 & -6.2282 & -1.1932 \end{bmatrix}, \tag{14}$$

the linear system (10) was obtained by solving a LQR problem

$$K_e = \begin{bmatrix} -2.9168 \\ 0.3185 \\ -18.6506 \\ 7.7213 \end{bmatrix} \tag{15}$$

and the lens gain

$$K_{lens} = 1000. \tag{16}$$

We have used Simulink™ and MATLAB™ to simulate the full dynamic motion of the Pendubot. Figure 4 shows the trajectory of the target in the image plane with ${}^{i}x_{d} = 0$, for convenience.

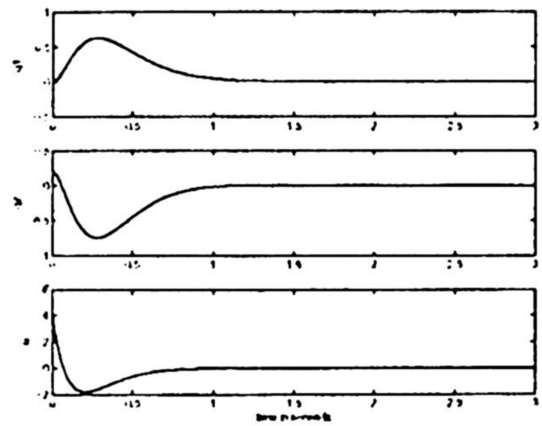

Fig. 4. Simulation results. Positioning with respect to the target Xt = (0; 90_).

5 Conclusions

The control of the Pendubot is specially dificult since it is an underactuated mechanism (two degrees of freedom and only one input). In this work, we have presented a linear position tracking controller for a fixed camera vision-based, Pendubot system. Specifically, by assuming exact knowledge of the mechanical parameters, and by considering an accepted camera model together with the robot linear dynamics, we have proposed an image-based visual servoing scheme derived from based on the combination of an estimator and the visual feedback, the design shows that this controller provides a good performance when balancing the links about the unstable vertical position. Numerical simulations assuming a discrete-time implementation of the visual controller showed the performance of the closed loop system. Preliminary results indicate that visual servoing is potentially attractive alternative for underactuated systems. We are currently implementing the algorithm on a Texas Instruments TMS320C6711 digital signal processor based system.

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